P P SAVANI UNIVERSITY

Third Semester of B. Tech. Examination

Nov-Dec 2021

SESH2031 Differential Methods for Chemical Engineers

03.12.2021, Friday **Time: 09:00 a.m. To 11:30 a.m.** **Maximum Marks: 60**

Instructions:

1. The question paper comprises of two sections.
2. Section I and II must be attempted in separate answer sheets.
3. Make suitable assumptions and draw neat figures wherever required.
4. Use of scientific calculator is allowed.

SECTION – I

Answer the Following: (Any Six)

Q - 1 Solve $y \log y \, dx + (x - \log y)dy = 0$. **[05]**

Q - 2 Solve $\frac{dr}{d\theta} = r \tan\theta - \frac{r^2}{\cos\theta}$. **[05]**

Q - 3 Using the method of variation of parameters, solve $(D^2 - 2D + 2)y = e^x \tan x$. **[05]**

Q - 4 Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$, given that $\frac{\partial z}{\partial y} = -2 \sin y$ when $x = 0$ and $z = 0$, when y is an odd multiple of $\frac{\pi}{2}$. **[05]**

Q - 5 Solve $(D^2 - 2DD' + D'^2)z = \tan(y + x)$. **[05]**

Q - 6 Solve $(y + z)p + (z + x)q = x + y$. **[05]**

Q - 7 Find the Laplace transform of the following: **[05]**

(a) $(1 + 2t - 3t^2 + 4t^3)u(t - 2)$

(b) $\int_0^t \int_0^t \sin at \, dt \, dt$

Q - 8 Find the inverse Laplace transform of $\frac{s-3}{s^2+4s+13}$. **[05]**

Q - 9 Find the inverse Laplace transform of $\frac{1}{(s-2)^4(s+3)}$ using convolution theorem. **[05]**

SECTION – II

Q - 1 Answer the Following: **[05]**

(i) Z-transform of 4^n is _____

(a) $\frac{z}{z+4}$ (b) $\frac{z}{z-4}$

(c) $\frac{1}{z+4}$ (d) $\frac{1}{z-4}$

(ii) Which of the following is an odd function?

(a) $\sin x$ (b) $e^x + e^{-x}$

(c) $e^{|x|}$ (d) $\pi^2 - x^2$

(iii) Fundamental period of $\sin 2x$ is ____

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$

(c) 2π (d) π

(iv) A function $f(x)$ is said to be even if

(a) $f(-x) = f(x)$ (b) $f(-x) = -f(x)$

(c) $f(x \pm 2\pi) = f(x)$ (d) $f(-x) = (f(x))^2$

(v) The value of a_0 in Fourier series expansion of $f(x) = x^2, -1 < x < 1$ is _____

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$

(c) 3 (d) 1

Q - 2 (a)	Find the Z-transform of $n\cos n\theta$	**[05]**
Q - 2 (b)	Find the half-range cosine series of $f(x) = (x-1)^2$ in $0 < x < 1$.	**[05]**

OR

Q - 2 (a)	Find the Z-transform of $\frac{7z-11z^2}{(z-1)(z-2)(z+3)}$	**[05]**
Q - 2 (b)	Find the half-range sine series of $f(x) = x^2$ in the interval $(0, \pi)$.	**[05]**
Q - 3 (a)	Find the Fourier series of $f(x) = x$ in $-\pi < x < \pi$.	**[05]**
Q - 3 (b)	Find the Fourier integral representation of the function $f(x) = \begin{cases} 1-x^2, & \lvert x\rvert \le 1 \\ 0, & \lvert x\rvert > 1 \end{cases}$	**[05]**

OR

Q - 3 (a)	Find the Fourier series of $f(x) = e^{ax}$ in the interval $(-\pi, \pi)$.	**[05]**
Q - 3 (b)	Find the Fourier cosine integral of $f(x) = \frac{\pi}{2}e^{-x}, x \ge 0$	**[05]**
Q - 4	Attempt any one:	**[05]**
(ii)	Using Fourier transform, find the solution of the differential equation $y' - 4y = H(t)e^{-4t}, -\infty < t < \infty$ where $H(t) = u_0(t)$ is the unit step function.	
(iii)	Use convolution theorem to evaluate $Z^{-1}\left\{\frac{z^2}{(z-a)(z-b)}\right\}$.	

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